

# Modelling Elections

Submission to ERRE: Special Committee on Electoral Reform

Byron Weber Becker

[bwbecker@golden.net](mailto:bwbecker@golden.net)

## Summary

I have done extensive computer modelling of six different electoral systems being proposed for Canada. The detailed results inform five recommendations regarding the systems. The two most prominent are that (1) Alternative Vote violates the Effectiveness and Legitimacy test in the Committee's mandate and (2) the Rural-Urban PR model advanced by Fair Vote Canada is highly proportional and has objective advantages relative to Single Transferable Vote and Mixed Member Proportional.

When I was eighteen my family moved and needed a new home. We decided to build one of Buckminster Fuller's geodesic domes – five-eighths of a sphere made from triangles with an unusual use of space inside the home. We decided on a dome in spite of the fact that we had only seen pictures of such a home. This lack of experience was a problem when faced with the decision of whether to build a massive, 7-meter-tall fireplace (at considerable expense) as a focal point in the middle of the dome.

The answer became clear when we built a scale model of the dome out of cardboard with a removable fireplace. We could get a preview of how the home would feel with the fireplace and without the fireplace. As a result of this modelling, the fireplace was built and we were delighted with the result.

Canada is now faced with designing something we have little direct experience with and considerably more complex than a geodesic dome. I think modelling the various alternatives will help guide us to better decisions.

Electoral systems can't be modelled out of cardboard, but they can be modelled with a computer. I have combined my vocation (teaching computer science at University of Waterloo) with my avocation (electoral reform) to do just that. I have written a computer program that takes the riding-by-riding results of the 2015 federal election and determines winning parties using different electoral systems, possibly after combining existing ridings into new ridings or adding top-up regions. More methodological details are included in Appendix A.

The electoral systems my program models are:

- Alternative Vote
- First-Past-The-Post
- Mixed Member Proportional (six variants)
- Single Transferable Vote (eight variants)

- Rural-Urban PR (twelve variants)
- and a Kingsley-like system (four variants)

By using a computer program rather than the typical spreadsheet, I am able to produce detailed statistics by region, estimate over- and under-representation by party, measure overall proportionality via Gallagher scores, calculate average riding sizes, and more. I believe these are the most detailed models describing possible Canadian electoral systems that are available today.

The results of my simulations are published at <http://election-modelling.ca>.

In addition to the primary simulations using 2015 federal election data, I also run ten additional simulations for each system in which votes are swung from one party to another in various degrees to verify that the system accurately tracks voter intention across a range of election scenarios.

Of the 32 electoral system variants that I've modelled, the nine that I consider most interesting are summarized in Figure 1. In the following subsections, I offer comments on these systems. Most of the comments concern the Committee's first criteria, Effectiveness and Legitimacy, which focuses on fairness and proportionality – that the “democratic will” of Canadians be “fairly translated” with “reduced distortion” in the election of representatives.

This chart, and especially the two graphs that follow, are easiest to interpret if viewed in colour. They can be viewed in colour, if necessary, at [http://election-modelling.ca/ModellingElections\\_en.pdf](http://election-modelling.ca/ModellingElections_en.pdf).

System Name	Number of Local MPs [1]	Number of Regional MPs [2]	Over-Representation by Party [3]					Gallagher Index [4]			% Voters with Preferred [5]		Short System Name [9]	
			Liberal	Conservative	NDP	Bloc	Green	2015 [5]	Composite [6]	Local MP [7]	Regional MP [8]			
Alternative Vote	338	0	24%	-11%	-4%	-4%	-3%	19.0%	24.0%	48%	48%	FP-A-x		
First-Past-The-Post	338	0	15%	-3%	-7%	-2%	-3%	12.0%	17.1%	48%	48%	FP-F-x		
Mixed Member Proportional (Lite)	338	53	11%	-2%	-5%	-1%	-3%	9.0%	11.4%	48%	74%	ME-F-x		
Mixed Member Proportional (Small Regions)	212	126	5%	-1%	-0%	-0%	-3%	3.8%	4.7%	47%	89%	MS-F-x		
Rural-Urban PR (More Singles, 389 Seats)	338	51	3%	1%	-0%	-1%	-2%	3.1%	4.1%	80%	92%	RE-F-S		
Rural-Urban PR (Few Singles)	338	52	3%	-1%	-0%	-0%	-1%	2.4%	3.3%	90%	96%	RM-A-S		
Rural-Urban PR (More Singles, 338 Seats)	287	51	3%	-1%	1%	-1%	-2%	2.7%	3.4%	80%	94%	RS-A-L		
Single Transferable Vote (Medium-sized Regions)	338	0	4%	-0%	0%	-0%	-3%	3.3%	4.2%	96%	96%	SM-A-S		
Single Transferable Vote (Small Regions)	338	0	7%	-1%	-2%	-1%	-3%	5.6%	6.5%	90%	90%	SS-A-S		

Figure 1: Summary of nine electoral systems' properties

Explanation of Figure 1 columns:

1. **Number of Local MPs** is the total number of MPs representing specific ridings. Those ridings may be either single-member ridings or multi-member.
2. **Number of Regional MPs** is the total number of MPs that represent multiple ridings. This happens in systems with top-up seats such as MMP and RU-PR.
3. **Over-Representation by Party** is the percentage of MPs in Parliament minus the percentage of the popular vote. For example, in 2015 under FPTP the Liberals received 54.4% of the seats but only 39.5% of the vote for an over-representation of  $(54.4 - 39.5) = 14.9\%$ . Negative numbers mean the party was under-represented.
4. **Gallagher Index** is a measure of disproportionality. It combines both over and under-representation for each party into a single number. Gallagher indices less than 5 are excellent.
5. **Gallagher Index 2015** is the Gallagher Index for the simulated 2015 election.
6. **Gallagher Index Composite** is the average of the Gallagher Indices for each province and territory, weighted by its number of seats. This corrects for a problem in calculating the Gallagher Index for the nation as a whole, which can hide regional disproportionalities such as the significant over-representation of Conservatives in the Prairies offsetting the over-representation of Liberals in the Maritimes.
7. **% Voters with Preferred Local MP** is the percentage of voters who have an MP representing their riding from the same party as their first choice candidate. Systems with multi-member ridings will do better under this measure.
8. **% Voters with Preferred Regional MP** is the percentage of voters who have an MP representing their region from the same party as their first choice candidate. Systems with top-up seats will do better under this measure.
9. **Short System Name** is a very concise abbreviation of the key parameters for this simulation.

## First-Past-The-Post

First-Past-The-Post (FPTP) is widely viewed in Canada as a problem. It over-represented Liberal voters by 15% and under-represented everyone else in 2015. In 2011 it over-represented the Conservatives by the same degree. In the 2015 Alberta provincial election FPTP over-represented the NDP. Fully 50% of Canada's elections since 1930 have resulted in false majorities in which the winning party received a Parliamentary majority they did not deserve.

Figure 1 shows the over- and under-representation of voters by party. For example, FPTP over-represents Liberal voters by 15% while under-representing Conservatives and NDP voters by 3% and 7%, respectively. These values are succinctly captured by the Gallagher Index (12.0%). This index (developed by Michael Gallagher, who testified to ERRE July 26, 2016) measures the overall disproportionality of an election result. The Gallagher Index for FPTP is the third worst of the 32 system variations I simulated.

Many observers have pointed out that Alberta, for example, is over-represented by Conservatives while Toronto and the Maritimes are over-represented by Liberals. The mathematics of the traditional Gallagher Index doesn't account for these regional disparities. The Composite Gallagher Index does; with this index, FPTP's value (17.1%) is 31<sup>st</sup> out of the 32 systems modelled.

Finally, the graph in Figure 2 is admittedly complicated but it shows a very simple truth: as the dominant party gains more votes, they gain seats at an even faster rate. For example, the point marked “A” says that if the Liberals were able to attract 10% of the Conservative vote in every riding, their share of the MPs would have increased from 54.4% (point “C”) to 64.5%, even though their total number of votes (point “B”) would have only grown from 39.5% (point “D”) to about 42.7%. In a perfectly proportional system the line with point “A” would be so close to the line with point “B” that you couldn’t tell them apart.

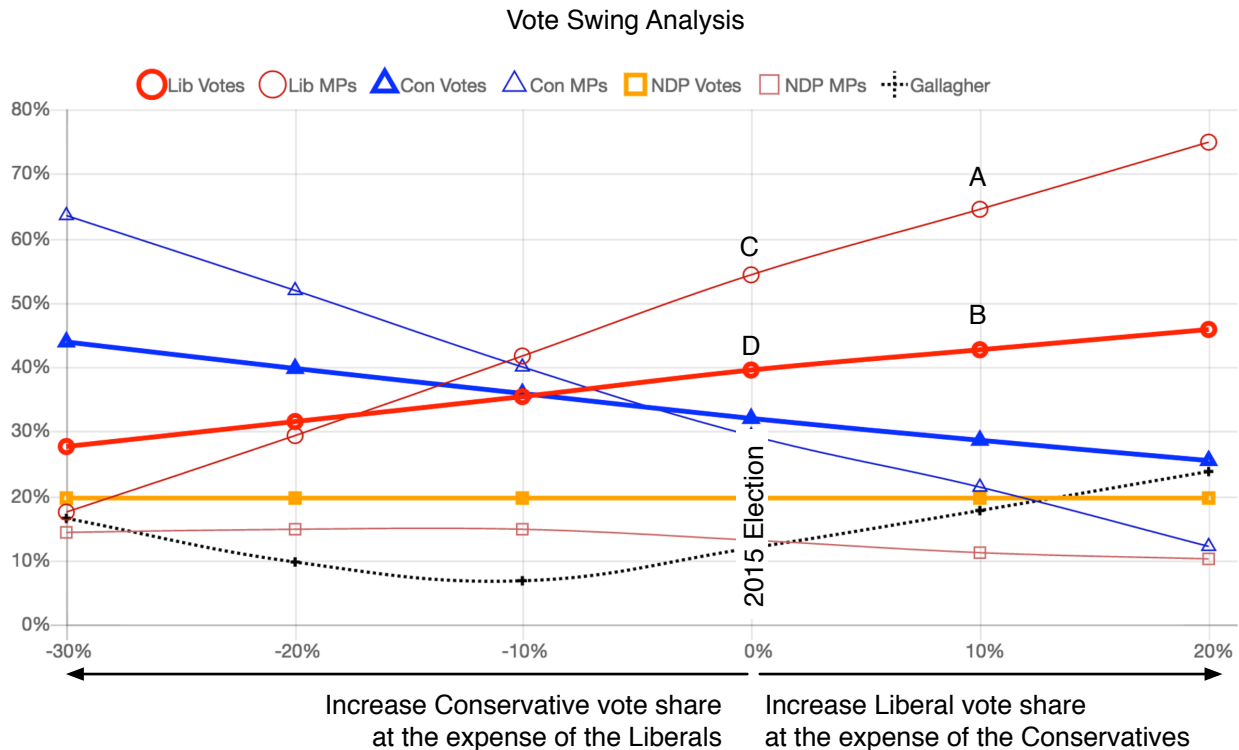


Figure 2: First-Past-The-Post Vote Swing Analysis

### Alternative Vote

The Alternative Vote (better named Instant Runoff Voting) is attractive to many Canadians. I used to advocate for it myself, but now consider it to be an example of The Tragedy of the Commons. In that economic example, the self-interest of individuals works against the best interest of the entire community.

Advocates of Alternative Vote are acting in the self-interest of each individual riding. AV tends to yield a compromise candidate that is acceptable, at some level, to a majority of the riding’s voters<sup>1</sup>. But just like the Tragedy of the Commons, the cost to the entire community – Canada as a whole – is huge.

<sup>1</sup> But not always! Sometimes what would be the consensus candidate is dropped early and voters are left with one of the extremes. Furthermore, it can deliver quite unintuitive results. See <http://zesty.ca/voting/sim/> for simulations at the individual riding level.

My simulations indicate that the 2015 election under AV would have increased the Liberal over-representation from 15% to 24%. They would have gained 63% of the MPs on only 39.5% of the first choice ballots. The Composite Gallagher Index of 24% is the worst of any system I simulated. As a result, I believe the adoption of AV would be a fundamental violation of the committee's first principle of Effectiveness and Legitimacy.

### Rural-Urban Proportional Representation

Based on my simulations, I believe that Rural-Urban Proportional Representation is the best system for Canada. RU-PR was designed by Fair Vote Canada and is described in their submission to the Committee.

RU-PR's proportionality is excellent. In all RU-PR variants the Gallagher Index is 4.1% or less and usually less than 3%. Furthermore, the Composite Gallagher Index is also low, indicating that it is very proportional in individual regions. This is confirmed by the detailed region-by-region analysis available on the web site (for example, <http://election-modelling.ca/rup-338/regionResults.html>). It's hard to find an STV or MMP simulation that gives better results.

In Figure 2 we saw that FPTP becomes even more disproportional as the leading party receives more votes. Figure 3 shows that Rural-Urban PR remains proportional through a wide range of voting scenarios.

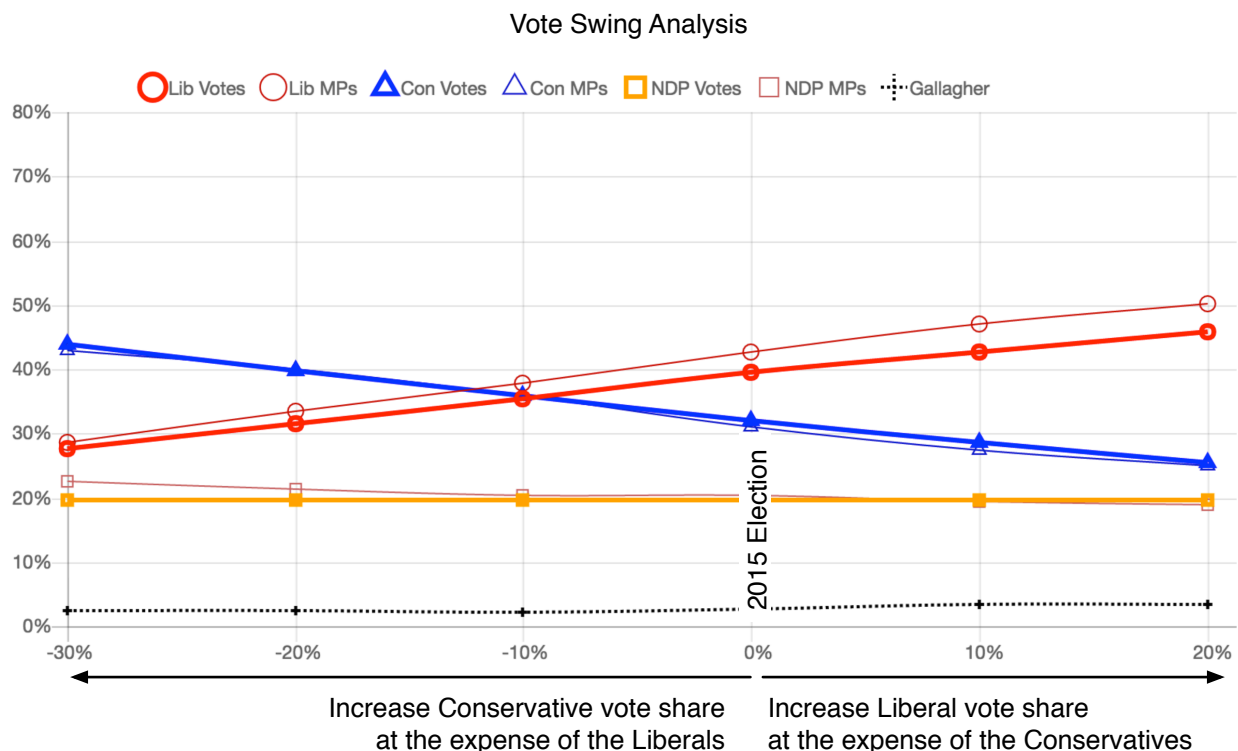


Figure 3: Rural-Urban Proportional Vote Swing Analysis

It is remarkable that RU-PR is able to achieve this level of proportionality with only 15% top-up seats. That compares very favourably to the 37% top-up seats often suggested for MMP. A practical result of that difference is that each local RU-PR MP represents an average 116,000 people whereas a local MMP MP represents an average 158,000 people.

RU-PR compares favourably to STV in terms of riding sizes. STV's use of multi-member ridings (nearly) everywhere would result in half of Canada's population living in ridings that are larger than 26,400km<sup>2</sup>. Meanwhile, under RU-PR half of our population would live in ridings that are less than 3,000km<sup>2</sup>.

### Single Transferrable Vote

I simulated Single Transferable Vote with small riding sizes (averaging 4 MPs/riding) as well as medium-sized ridings (averaging 11 MPs/riding). STV with larger ridings was quite proportional. The variant with smaller ridings over-represented Liberals by 7% and had a Gallagher score of 5.6%.

### Mixed Member Proportional

As with STV, I simulated two variations of Mixed Member Proportional: one with small top-up regions (8 MPs/region) and one with larger top-up regions (almost 13 MPs/region). Both showed very good proportionality when using FPTP elections in the local ridings. Using an AV-style election in the individual ridings compromised the proportionality. The variant with larger regions performed somewhat better.

### MMP-Lite

It might seem tempting to preserve our existing 338 single-member ridings while adding a measure of proportionality by enlarging Parliament with a small top-up layer of about 50 seats. I've called this option "MMP-Lite" in Figure 1. Simulations show that it is better than FPTP, but not by much. The Rural-Urban Proportional system does much better with a similar set of constraints.

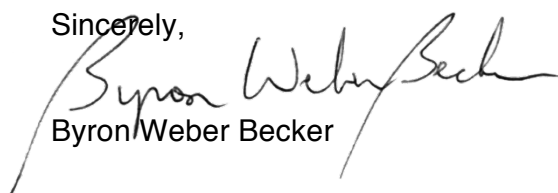
## Recommendations

With respect to the first principle of Effectiveness and Legitimacy, I recommend that

1. the Committee issue a preliminary report stating that the Alternative Vote would be a step backward from FPTP and should not be considered further;
2. the Committee strongly consider Rural-Urban PR, a highly proportional, made-in-Canada system that effectively deals with our diverse riding sizes;
3. if choosing STV, the Committee think carefully about whether having smaller multi-member ridings is worth the decreased proportionality;
4. if choosing MMP, the Committee should stipulate that FPTP (rather than AV) continues to be used in the local riding elections;
5. the Committee avoid MMP-Lite's substantial increase in complexity for very little gain in proportionality.

Thank you for considering my findings and recommendations. I would welcome further dialog, in person if appropriate, with the Committee.

Sincerely,



Byron Weber Becker

## Appendix A: Methodology

The sample input file shown in Figure 4 reveals much about the operation of the simulation program. This particular input file is for an MMP simulation.

Line 01 starts a list of Canada's provinces. Each province has one or more regions. Alberta's first region, AB.Calgary (line 06) has four top-up seats (line 07).

AB.Calgary has six new ridings, named "AB.Calgary.1", "AB.Calgary.2", etc. The first two start at lines 10 and 18.

The new riding of AB.Calgary.1 has a district magnitude of 1, meaning that 1 MP will be elected from that riding (line 11). In simulations with multi-member ridings the district magnitude may be larger.

AB.Calgary.1 includes 100% of the 2015 riding of Calgary Rocky Ridge and 67% of the 2015 riding of Calgary Nose Hill (lines 13 and 14). Farther down, at line 22, we see that the other third of Calgary Nose Hill is incorporated into the new riding of AB.Calgary.2.

To summarize, the input file organizes 2015 ridings into new ridings which may elect one or several MPs, depending on the district magnitude. Those new ridings are grouped into one or more regions, each of which may have zero or more top-up seats. One or more regions make up a province.

Each electoral system has an input file. Briefly, they are:

- First-Past-The-Post: new ridings are the same as old ridings, each with district magnitude 1. Each province has 1 region with no top-up seats.
- Alternative Vote: Same as FPTP.
- Mixed Member Proportional: 2015 ridings are combined into new, larger ridings to free up MPs to fill the top-up seats. Each of the new ridings has district magnitude of 1. The new ridings are grouped into regions, each with an appropriate number of top-up seats. Larger provinces have multiple regions; smaller provinces have only one.
- MMP-Lite: 2015 ridings each form one new riding with a district magnitude of 1. New ridings are formed into relatively large regions with a small number of top-up seats. Larger provinces have multiple regions; smaller provinces have only one.

```
01"provinces": [  
02  {  
03  "prov": "AB",  
04  "regions": [  
05  {  
06    "region_id": "AB.Calgary",  
07    "top_up_seats": 4,  
08    "new_ridings": [  
09    {  
10      "riding_id": "AB.Calgary.1",  
11      "district_mag": 1,  
12      "old_ridings": [  
13        "48009, 67, Calgary Nose Hill",  
14        "48010, 100, Calgary Rocky Ridge"  
15      ]  
16    },  
17    {  
18      "riding_id": "AB.Calgary.2",  
19      "district_mag": 1,  
20      "old_ridings": [  
21        "48006, 33, Calgary Forest Lawn",  
22        "48009, 33, Calgary Nose Hill",  
23        "48013, 100, Calgary Skyview"  
24      ]  
25    },  
    ...  
  ]  
04  },  
  ]  
]
```

Figure 4: Sample input file

- Single Transferable Vote: 2015 ridings are combined with adjacent ridings to form new, multi-member ridings. The new riding's district magnitude is equal to the number of ridings that were combined. New ridings are combined into a single region for the entire province with no top-up seats.
- Rural-Urban PR: Some 2015 ridings form new ridings all by themselves (the single-member ridings) while others are combined into new, multi-member ridings. District magnitudes vary between 1 and about 6, as appropriate. New ridings are gathered into regions, each with top-up seats. Large provinces have multiple regions.

After grouping the ridings and regions, as described above, the simulation program calculates the winners in each riding. It uses both First-Past-The-Post and Alternative Vote algorithms in single-member ridings. In multi-member ridings it uses Single Transferable Vote and List-PR algorithms. Each combination of input file, single-member riding algorithm and multi-member riding algorithm has its own set of outputs and is published at <http://election-modelling.ca>.

### Source Code

The source code and input files are available at <https://github.com/bwbecker/electionSim> so that anyone can examine it in detail, reproduce my results, or extend the results to still more electoral systems.